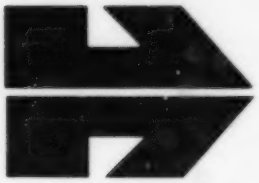

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WORKING PAPER NO. 13

AN ANALYSIS OF SNOW CLEARANCE ROUTES IN THE CITY OF NORTH YORK

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ABSTRACT

In the quest for more efficient operating procedures in the public service sector, efforts in the field of operations research are being directed at various aspects of the delivery of public services, such as waste collection, street cleaning, and snow removal. The routing of vehicles through city streets is one problem that is encountered in the analysis of public services, and the search for the shortest route is the subject of this paper.

First, two separate strategies for generating routes through a street network are reviewed. The two strategies are, first, a derivation of one of Euler's theorems, the Chinese Postman Problem, and secondly, intuition. Various problems encountered in the application of the Euler procedure and intuition are identified and discussed.

Second, a formal method combining Euler's procedure(s) and intuition for generating minimum-distance paths is developed and tested through experimentation using different groups of subjects.

The final section of this paper reviews the results of the previous sections and attempts to prepare a methodological basis for applying the ONLIP (Odd Node Linking and Intuitive Procedure) theory for solving real world vehicle routing problems in cities.

The major weaknesses of the method are identified in the paper. It is hoped that this work will contribute toward improving snow clearing effectiveness through the understanding of ways for tackling vehicle routing problems.

TABLE OF CONTENTS

List of Tables vi

List of Figures vi

I. A Comparison of Strategies for Generating Routes Through a Street Network

1.0 General Introduction 1

2.0 Solution Method for Edge Covering: The Chinese Postman Problem 2

2.1 Problems in Tracing a Path Through a Real Street Network 4

2.2 Euler Path Procedure and Intuition: Two Approaches for Solving the Vehicle Routing Problem 13

3.0 Conclusions 24

II. Euler's Procedure and Intuition as a Combined Formal Method for Solving Minimum-Distance Paths

1.0 General Introduction 26

2.0 Euler Procedure 27

3.0 Intuitive Procedure 30

4.0 Minimum-Distance Path Solutions 31

4.1 Objectives 31

4.2 Method 32

4.3 Subjects 32

4.4 Results of Experiments 33

4.5 Costs 37

5.0 Conclusions 38

III. Application of ONLIP to a Real Street Network

1.0 General Introduction 40

2.0 Actual Path 41

3.0 Application of ONLIP 45

References 46

Appendix 48

LIST OF TABLES

1.	Solutions for the Minimum-Distance Path Using Intuition for Inroad 4	21
2.	Duplication of Minimum-Distance Path (in Units) by Same Subjects with Successive Trials Using Intuition and With Euler's Theorem	34
3.	Duplication of Minimum-Distance Path of Different Subject Groups Using Intuition (in Units)	35

LIST OF FIGURES

1.	Seven-Bridge Problem	3
2.	Sample Network	5
3.	Districts for Snow Removal Operation in the City of North York	6
4.	Ingram District 6W (Inroads 4 and 5)	8
5.	Yellow Routes in Inroads 4 and 5	9
6.	Unmarked Routes in Inroads 4 and 5	10
7.	Red Routes in Inroads 4 and 5	11
8.	(Inroad 4) North York's Roads Only	15
9.	(Inroad 4) North York Roads and All Metro Roads	16
10.	Even and Odd Nodes in a Network	17
11.	Euler Network for Inroad 4	19
12.	Euler Path for Inroad 4	20
13.	Frequency Curve of Minimum-Distance Path Solution	23
14.	Different Routing Patterns	29
15.	Inroad 4 - Direction of Real Path	42
16.	Inroad 5 - Direction of Real Path	43

PART ONE:
A COMPARISON OF STRATEGIES FOR GENERATING ROUTES THROUGH A STREET NETWORK

1.0 GENERAL INTRODUCTION

A common feature in the provision of urban services such as street cleaning, waste collection and snow removal is that a vehicle has to travel along a street network in order to perform its collection or distribution function.

There are many ways in which a vehicle can trace a continuous path through a street network so that it travels down every street; in such a situation, however, it is desirable to follow the path through the network which covers each street once and once only. This path is then by definition the least-distance path (and by inference the least-cost path) . If this least-distance path cannot be traced, the next best path must be found. This "next best" path is the one which incurs the least possible distance penalty over the least-distance path in satisfying the condition that each street must be travelled at least once. The distance penalty will occur because some streets have to be travelled twice in completing the path through the network. (Transportation Development Agency 1975, p. 72).

There have been several attempts to optimize routes for the collection and disposal of municipal refuse. An example of a simulation model, as a tool which may be adapted by municipal engineers, for the design of vehicle routing schemes is presented in the Journal of the Sanitary Engineering Division. The interest in routing problems as they apply to the snow clearance operation does not appear to be as intense as that for waste collection and disposal. The reasons for the interest in the waste collection service can be explained by the routine nature, predictability and the perceived demand for this service. The snow clearance service, on the other hand, is very unpredictable—both temporally and spatially—and as a result,

the demand for this service is perceived differently. Simply put, the routing problem for snow clearance is more complex. As such, it is not surprising to find that the literature tends to avoid the snow clearance operation as a subject of study in favor of the waste collection operation. This paper provides one of the few examples of a study of the snow clearance operation with a focus on the vehicle routing problem.

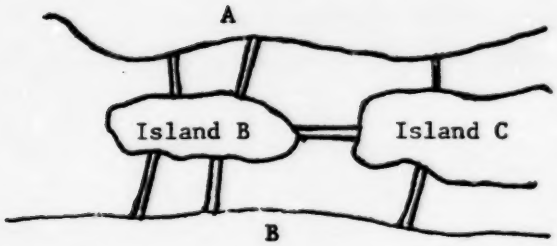
This section will first focus on a manual method--the linking of odd nodes and the derivation of an Euler path--as a solution method for routing a vehicle through a street network. Secondly, there will be a discussion of the problems and difficulties faced by the analyst in tracing a path for a small street network in the City of North York. Finally, there will be a discussion comparing the Euler path procedure with intuition as two approaches for solving the vehicle routing problem. Hence, the aim of this section is to provide some insights into the applicability and limitations in applying Euler's path and intuition in identifying the least-distance path for vehicle routing problems.

2.0 SOLUTION METHOD FOR EDGE COVERING: THE CHINESE POSTMAN'S PROBLEM

The edge covering problem or, as it is sometimes referred to, the Chinese Postman's Problem is to define a route that minimizes the total distance traversed by a vehicle, while at the same time traversing every street segment at least once. (An early paper discussing this problem appeared in the journal **Chinese Mathematics**, hence, the name.) Attempts to solve the Chinese Postman's Problem were first examined by a Swiss mathematician and physicist Leonhard Euler. In 1736 Euler tried to find a way in which a parade procession could cross seven bridges exactly once (see Fig. 1). In failing to find such a route, Euler derived a theorem which said that it is possible to draw a continuous path which covers each street once and only once, returning to the starting point if the number of odd nodes in the network is 0 or 2. In a street network, an odd node is the intersection of

Figure 1

SEVEN-BRIDGE PROBLEM



1, 3, 5, 7, etc. streets, and an even node is the intersection of 2, 4, 6, 8, etc. streets. Larson and Odoni (1981) present a detailed discussion, including sample graphs, to illustrate Euler's theorem (refer to pp. 386-90).

The Transportation Development Agency (1975) describes the solution method for dealing with a street network in three basic steps:

1. partition the network into manageable components;
2. link pairs of odd nodes, thus identifying streets which must be traversed twice;
3. solve the resulting network by using Euler's theorem.

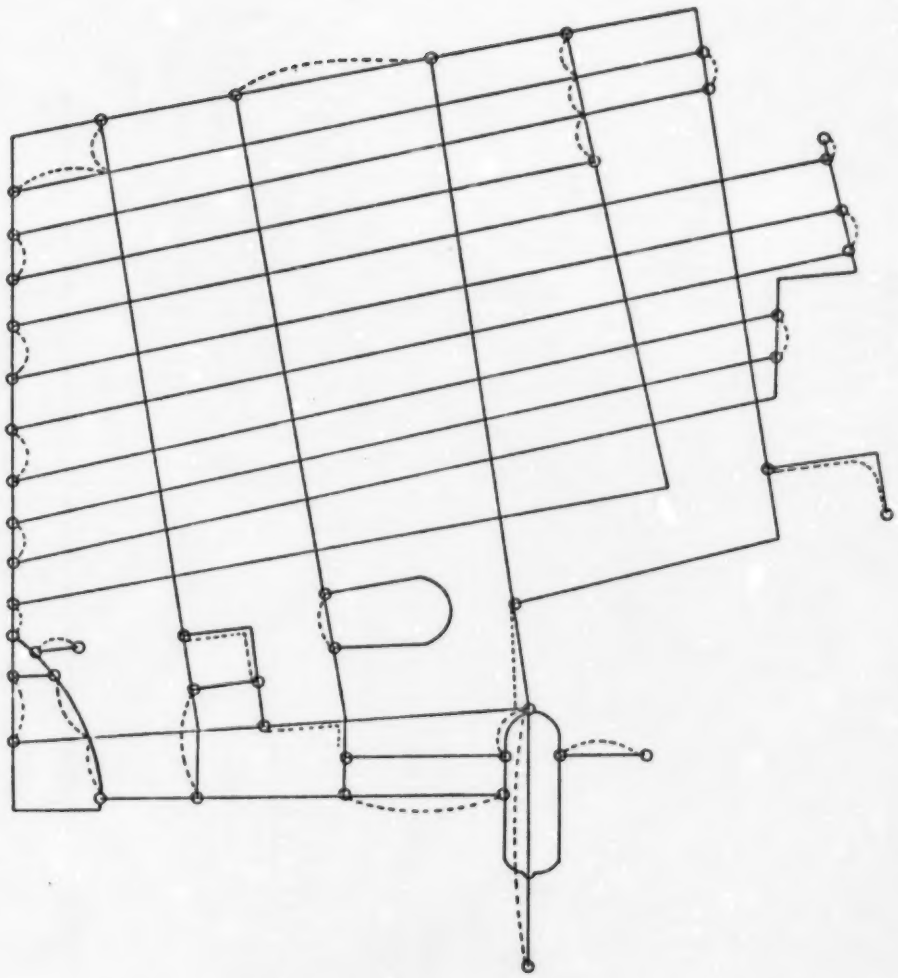
The solution method outlined above is a manual method which depends on the pattern recognition abilities of the analyst. It should be noted that there are degrees of complexity to the Chinese Postman Problem algorithm. Worthwhile discussions of the complexity of this algorithm are presented by Larson and Odoni (1981, pp. 376-379) and Edmonds and Johnson (1973, pp. 88-124).

One of the two sample networks included in the chapter on routing problems in the Transportation Development Agency report for which odd nodes have been circled and joined together in pairs by broken lines is represented by Fig. 2. The broken lines represent streets which have to be covered more than once. The corresponding Euler path for Sample Network No. 1 is derived by the author of this paper. The time taken to solve the network, including drawing the path, was less than one hour. An Euler's path derived for a small street network in the City of North York is discussed in the second section.

2.1 Problems in Tracing a Path Through a Real Street Network

The City of North York is divided into six snow removal districts and within each of these districts a number of route boundaries are identified (Refer to Fig. 3). Within each of these route boundaries a vehicle is

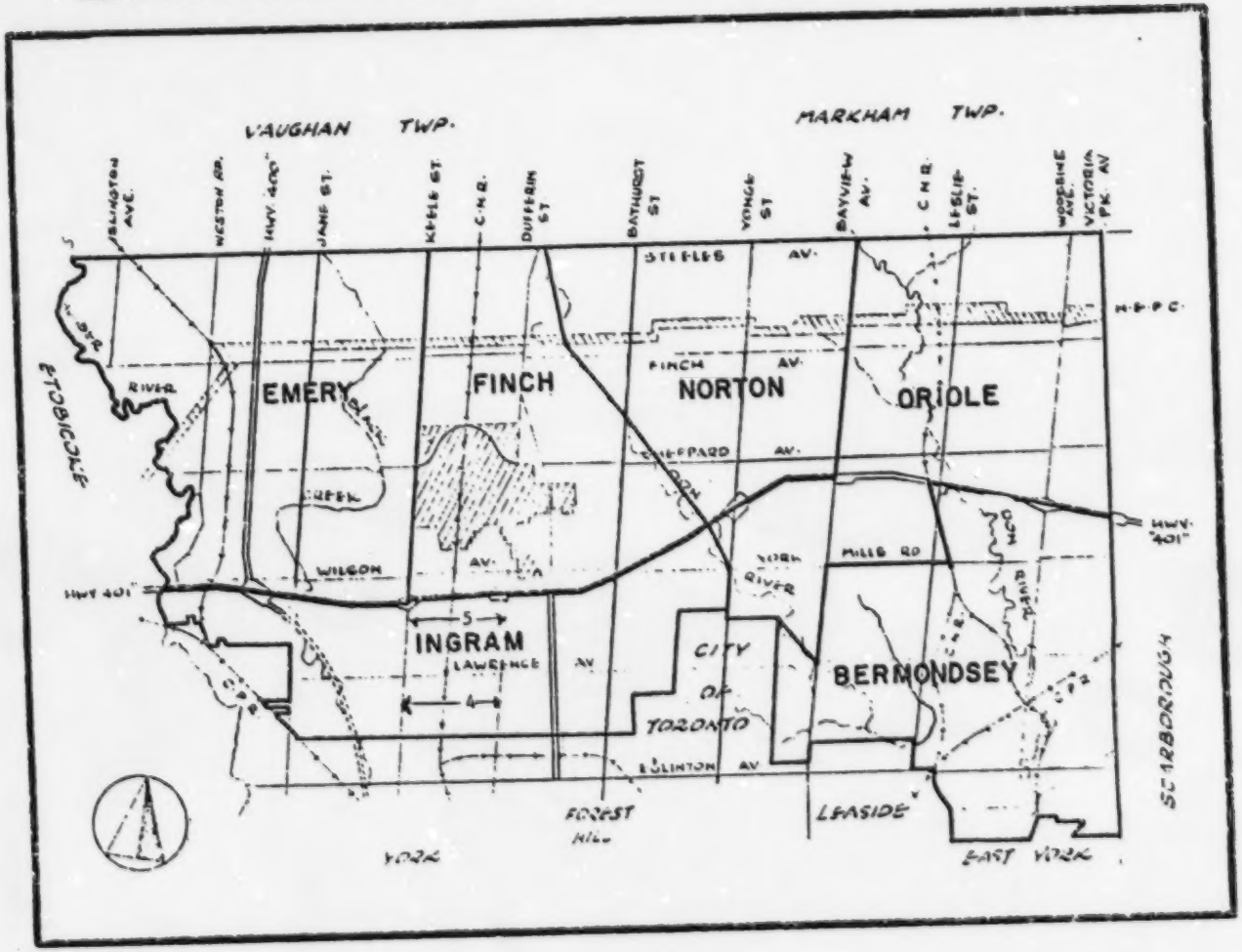
Figure 2
SAMPLE NETWORK



° Odd Node
--- Link to be Traversed Twice

Figure 3

DISTRICTS FOR THE SNOW REMOVAL OPERATION IN THE CITY OF NORTH YORK



assigned a specific route. Inroad 4 and Inroad 5 are two snow removal routes in Ingram District 6W which Beamish Contractors are responsible for clearing (refer to Fig. 4).

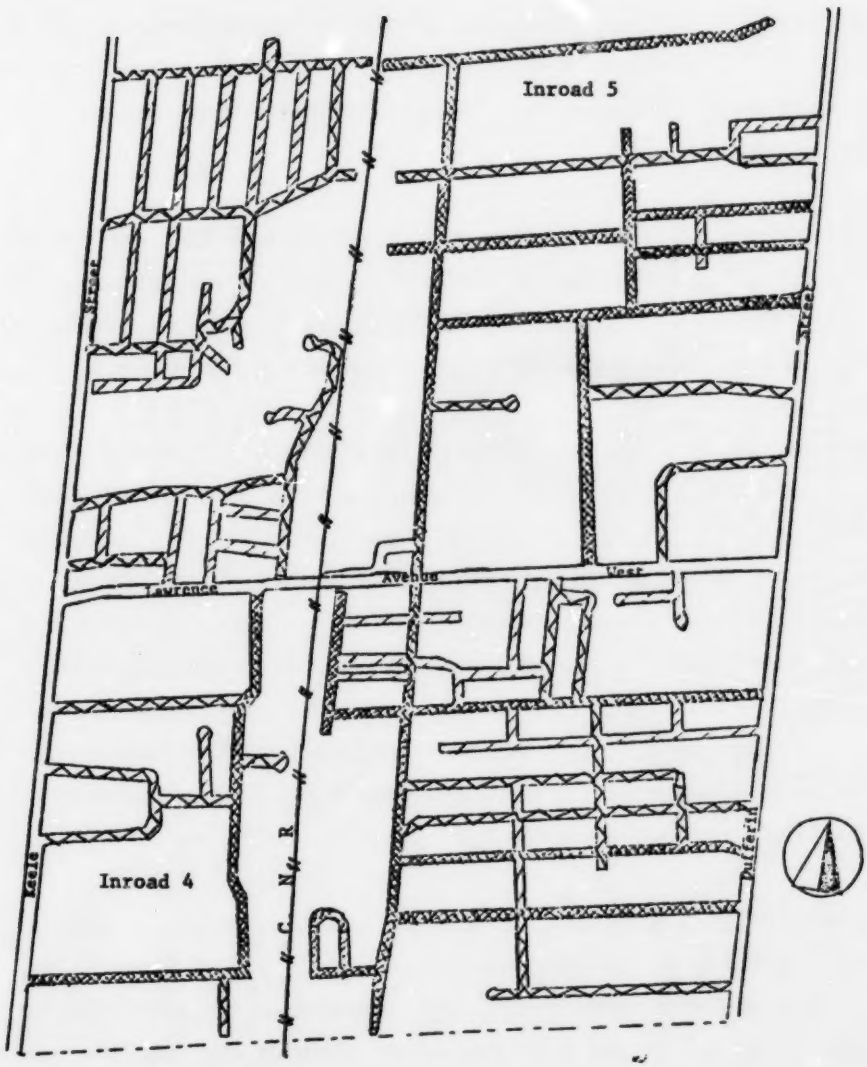
There are a number of differences between a sample network and a real network which must be taken into consideration when attempting to identify a path to be traversed by a vehicle. These differences may be summarized as follows:

1. **Assumptions** - Real street networks do not necessarily comply with the restraining assumption of the Euler method. The existence of 0 or 2 odd nodes and connectivity of the links are rarely found in real networks and certainly do not exist for Inroad 4 or for Inroad 5.

2. **Street Priority** - The City of North York identifies street priorities in their snowplow operation: (a) primary roads (red routes) which must be kept open at all times (i.e. bus routes, steep hills, collectors and traffic arteries); (b) secondary roads (yellow routes) where the centre portion is kept bare; (c) local roads (unmarked routes) which are cleared last. A street network that is cleared of snow according to street priority tends to produce disconnectivity of links, or it may amalgamate the different categories of street priority to minimize the total distance of the path and thus make it impracticable to apply the Euler method of tracing the minimum-distance path.

3. **Connectivity** - Unlike Sample Network No. 2, the road network for the City of North York does not have a high degree of connectivity. The lack of connectivity in the street network for the snow plowing route of Inroads 4 and 5 in the Ingram District can best be seen when the red, yellow and unmarked routes are individually represented (refer to Figures 5, 6 and 7). When no road priority exists, the street network has a higher degree of connectivity. The discontinuity of the links may remain if the path to be traversed is

Figure 4
INGRAM DISTRICT 6W



Red Routes (Primary Roads)
Yellow Routes (Secondary Roads)
Unmarked Routes (Local Roads)
Metro Roads

Figure 5

YELLOW ROUTES IN INROADS 4 AND 5

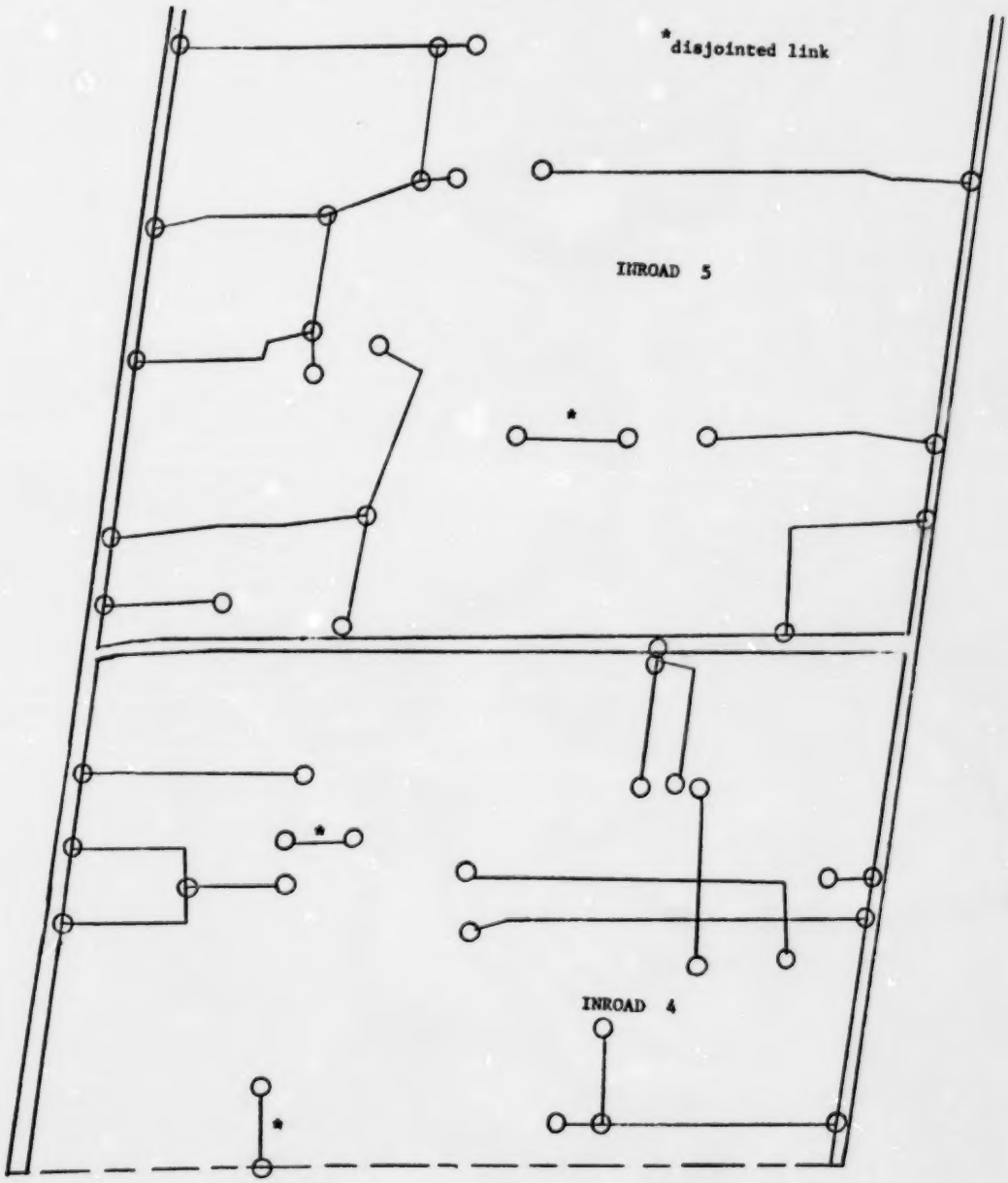


Figure 6

UNMARKED ROUTES IN INROADS 4 AND 5

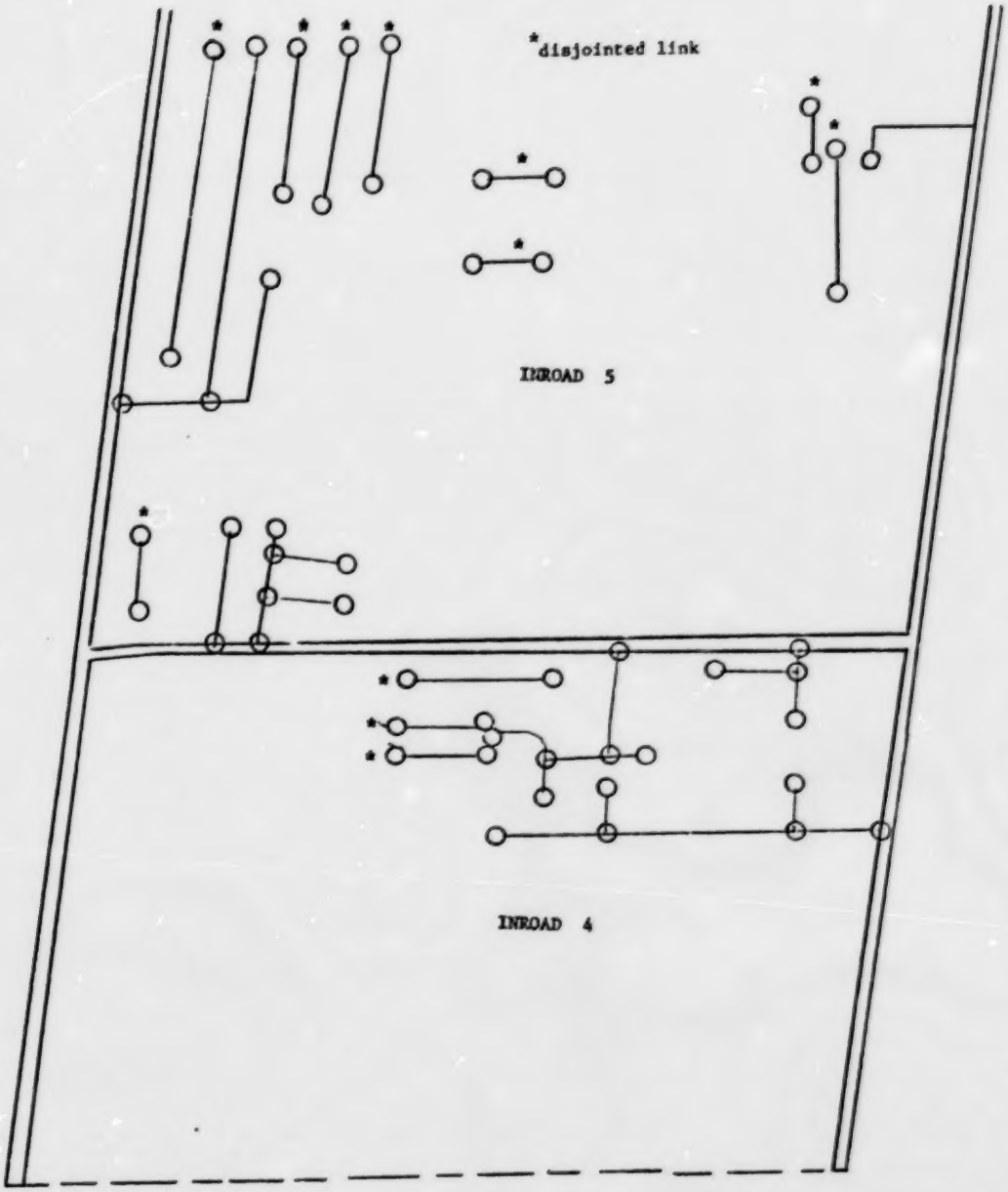
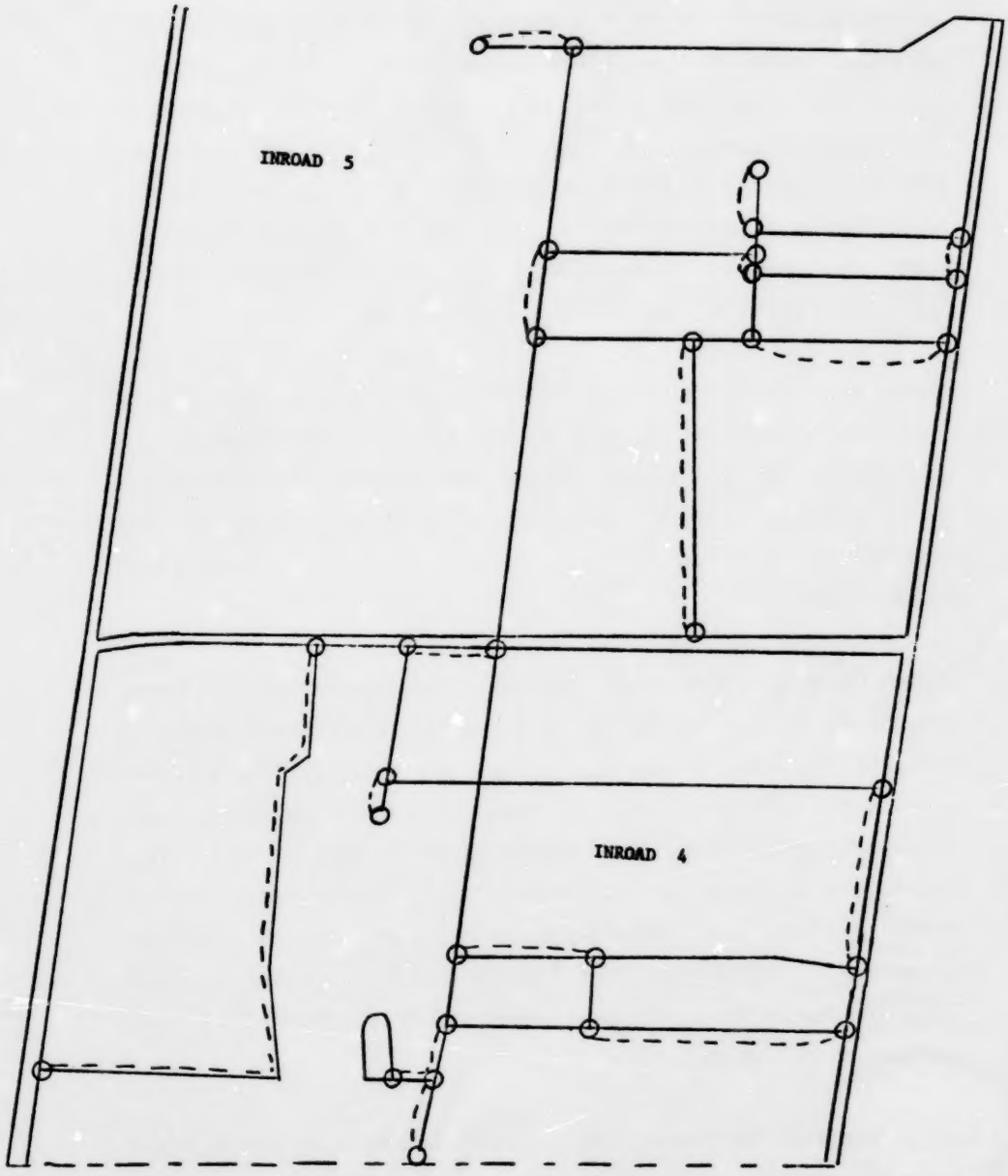


Figure 7
RED ROUTES IN INROADS 4 AND 5



divided by Metro roads (plowed by Metropolitan Toronto), CNR, parks, water bodies, large commercial centres (ie., Yorkdale Shopping Centre), and the like. The CNR in the snow removal route in Ingram District serves as a NS partition of the street network. Likewise, Lawrence Avenue West serves as an EW divider. In fact, for Ingram District, two of the ten snowplows owned by the Beamish Contractors are designated to snow removal route 6W--one snowplow is responsible for the street network south of Lawrence Avenue West (Inroad 4) and the other is responsible for the street network north of Lawrence Avenue West (Inroad 5)--and each vehicle approaches the street network from each side of the CNR.

4. **Direction** - Traffic direction and the fact that the blade of the snowplow is on the right side of the vehicle both influence the direction of the vehicle when tracing a path through the network. In a hypothetical network, traffic direction is not obeyed and no restrictions are placed on vehicle direction; in fact, this is an option not a reality.

5. **Street Coverage** - For a directed, real street network, both sides of the street are cleared (that is, the street is traversed twice, in opposite directions) whereas, in the sample network the path to be traversed is undirected and each street is to be traversed once and only once, if possible. The designated North York snow removal routes are enclosed by Metro roads such as Lawrence Avenue, Dufferin Street and Keele Street. These arterial roads are the responsibility of Metropolitan Toronto but have to be traversed when the North York roads are covered so that the North York roads are not discontinuous.

6. **Snow Removal Policies** - When a path is traversed in a real street network, certain policies of the snow removal operation are exercised, whereas in a hypothetical street network factors that affect

the snowplow operation are kept constant. The policies referred to are, for example, plowing always to the right, minimizing number of left turns at intersections, backing-up to clear streets, maximizing right turns, not travelling a straight path so that snow does not build up on the sides of the street, beginning road plowing when snow build-up is two inches and still falling, etc.

In attempting to use intuition as a procedure for identifying a path through a network, those street networks for Inroads 4 and 5, individually represented by the yellow and unmarked routes are discarded because of the discontinuity of the links (refer to Figures 5 and 6). Where there is a link that is not connected to any other link, it is inaccessible to the vehicle because it is disjointed from the links that are part of the path. These types of links are identified in Figures 5 and 6 by an asterisk (*). The red routes in Inroads 4 and 5 do not have any disjointed links for the street network (refer to Fig. 7). For these red routes a shortest-path can be traced--either by intuition or by applying Euler's theorem--because there are no disjointed links.

2.2 Euler Path Procedure and Intuition: Two Approaches for Solving the Vehicle Routing Problem

A street network for Inroad 4 is solved for the vehicle routing problem using intuition, represented by six subjects, and by applying Euler's path procedure. The resulting total distances travelled for the paths through the street network using the two approaches are calculated and compared. The two approaches for solving the vehicle routing problem are analyzed with respect to their applicability and limitations.

When tracing a Euler's path for Inroad 4, Metro roads are either to be excluded, or partly excluded, or excluded altogether from the street network to be traversed. The reason these three choices arise is because the City of North York is not responsible for clearing snow on Metro roads, but the snowplow can travel on these roads in order to clear the North York roads.

The street network for the snow plowing route (Inroad 4) becomes disconnected into two separate street networks (with the CNR as the divider) when Metro roads (Keele Street and Lawrence Avenue West) are excluded entirely from the street network. The total distance to be traversed when Metro roads are entirely excluded is 15.9 miles and the number of odd nodes produced are 60 (refer to Fig. 8). In addition, the North York streets become disjointed, thereby allowing for more odd nodes and, by inference, the transversal of more North York streets more than once.

When all three Metro roads are included in the street network both the number of odd nodes and the total distance of the path to be traversed through the network change slightly: there are 56 odd nodes and the street network to be traversed is 16.1 miles (refer to Fig. 9).

Thus, a decision must be reached regarding which sections of the Metro roads will reduce the number of odd nodes (and by inference the traversing of a link more than once) and total distance of the path.

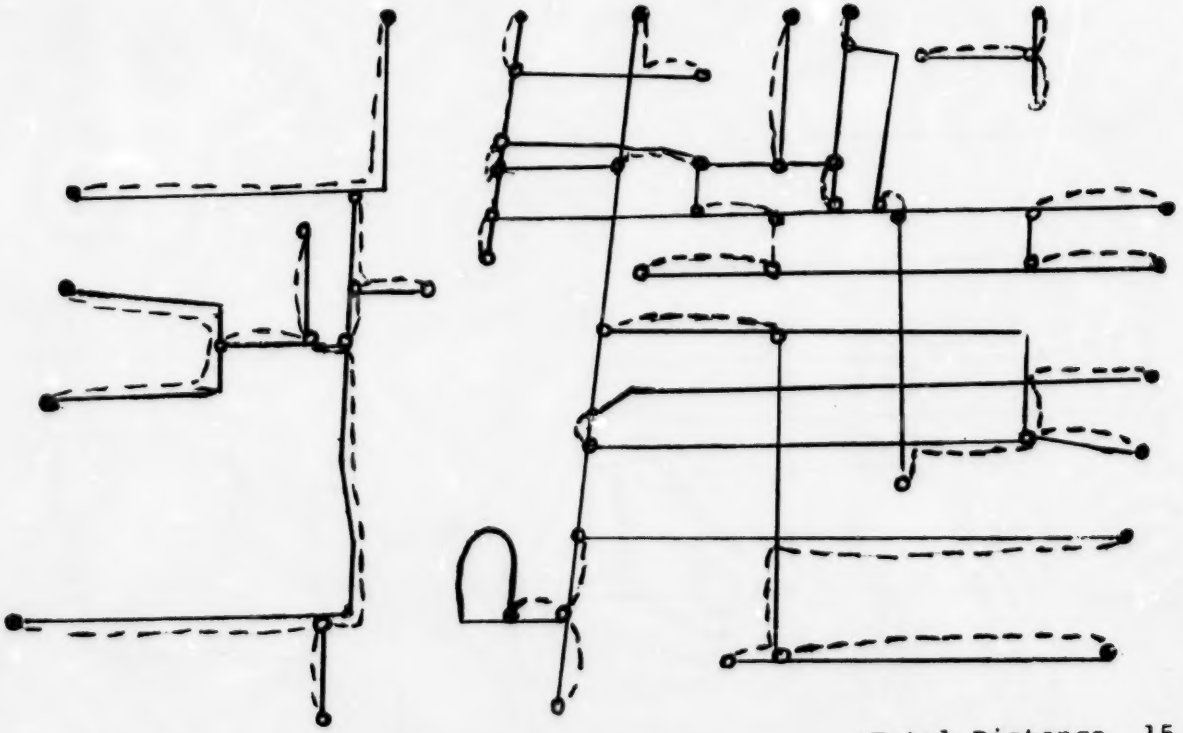
When the Metro roads are optional routes, the analyst uses his or her discretion for including or excluding Metro roads; it may be that nodes previously classified as odd nodes become even nodes or vice-versa (refer to Fig. 10).

The solution method for identifying a Euler path requires a street network with linked pairs of odd nodes already identified. The implication here is that the analyst cannot change an intersection from an odd to an even node or vice versa while identifying the least-distance path through the street network.

The objective of the analyst is to include Metro roads in the street network only when Metro roads either minimize total distance of the path of the vehicle, or when the number of odd nodes are reduced and by inference

Figure 8

(INROAD 4) NORTH YORK ROADS ONLY



o Odd Nodes

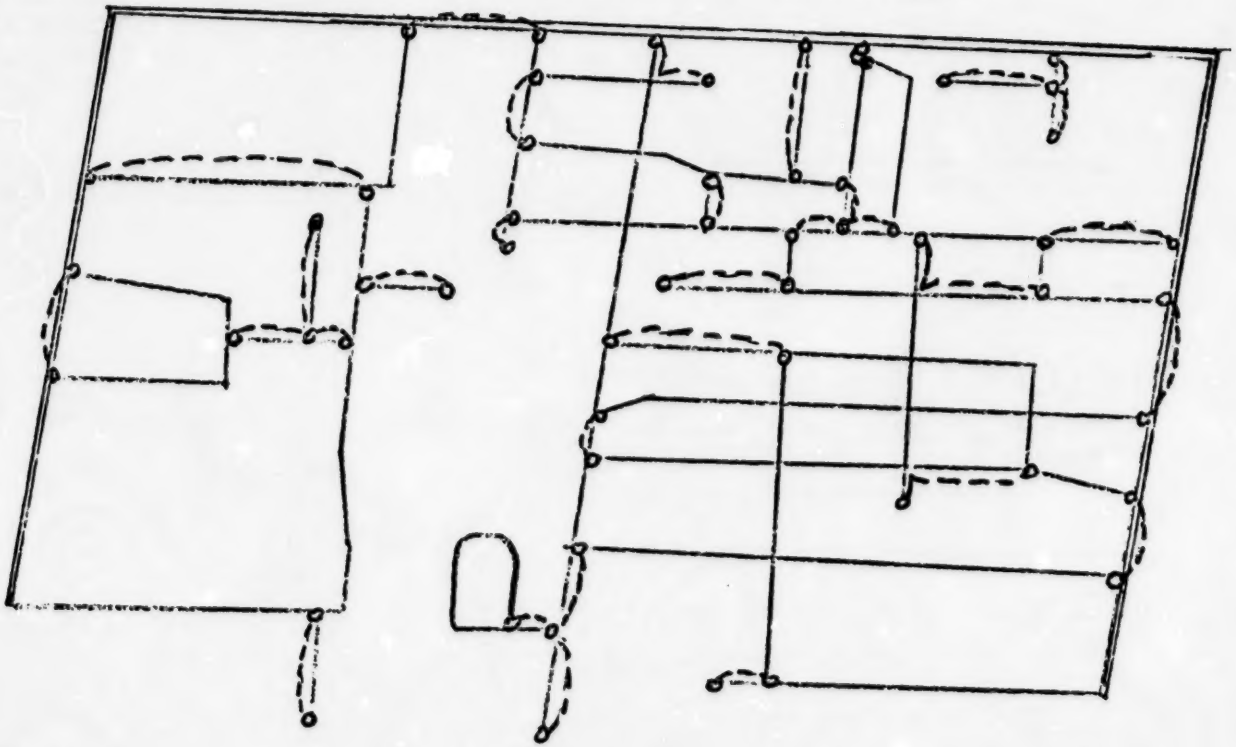
• Odd Nodes Created by Excluding Metro Roads from Network

Total Distance 15.9 Miles

No. of Odd Nodes 60

Figure 9

(INROAD 4) NORTH YORK ROADS AND ALL METRO ROADS



o 56 Odd Nodes

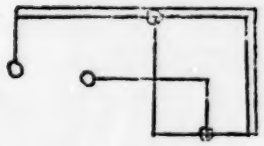
___ North York Roads

═══ Metro Roads

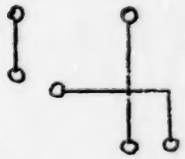
Total Distance 16.1 miles

Figure 10

EVEN AND ODD NODES IN A NETWORK



With Metro Roads (4 Odd Nodes)



Without Metro Roads (6 Odd Nodes)

═══ Metro Road
——— North York Road
o Odd Node

minimize the links that have to be covered more than once. Such a network is derived (refer to Fig. 11); the total distance for this network is 14.2 miles and the number of odd nodes produced is 46.

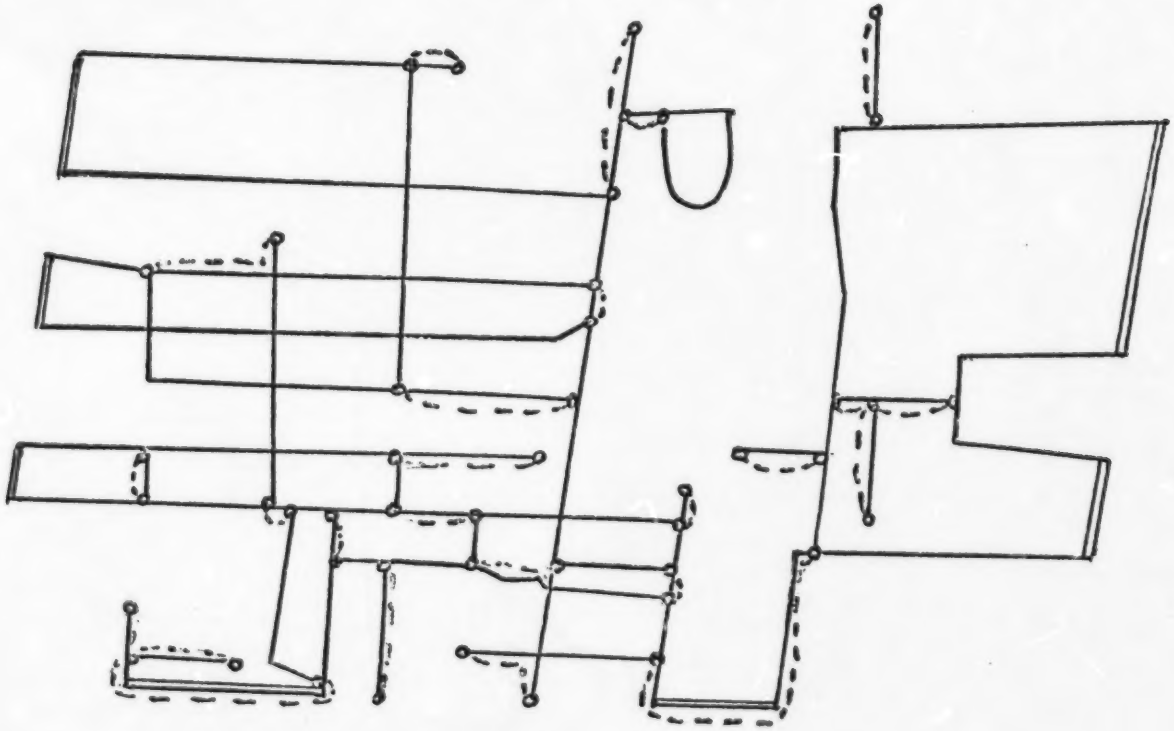
There are two problems with this Euler network. One problem is that the network for Inroad 4 is not an ideal Euler network, because it has 46 odd nodes as opposed to 0 or 2 odd nodes. Thus, a continuous path may or may not be possible through the network that duplicates those streets identified by the pairing of odd nodes without any other extra duplication. For the Euler network for Inroad 4 a Euler path is identified by the author (refer to Fig. 12) with no additional duplicated streets being covered. Perhaps, by either linking the odd nodes differently, or by an attempt to find a Euler path by another analyst, different results would emerge. A second problem is that the pairing of odd nodes according to the 'nearest odd node' rule is not always possible. That is to say, not all the odd nodes in Fig. 11 are linked on the basis of the nearest neighbouring odd node. The street pattern, the location and number of odd nodes as well as the analyst's ability to link pairs of odd nodes efficiently may account for difficulties in complying with this rule.

The Inroad 4 street network, including all North York and Metro roads, was given to six subjects in order to examine the minimum-distance values for the path through the street network using only intuition. The subjects were instructed that every link in the North York street network had to be traversed, but it was optional to traverse the Metro roads in an attempt to use intuition to minimize the total distance of a path through the street network. The subjects who participated in this experiment were graduates whose previous studies were related to geography. Three of the six subjects also had previous experience with street-network-type problems.

From the six solutions of minimum-distance paths identified using intuition, the lowest value for the total distance of the path is 14.3 miles and the largest value is 17.1 miles (refer to Table 1). The total distance of the

Figure 11

EULER NETWORK FOR INROAD 4



Total Distance 14.2 miles

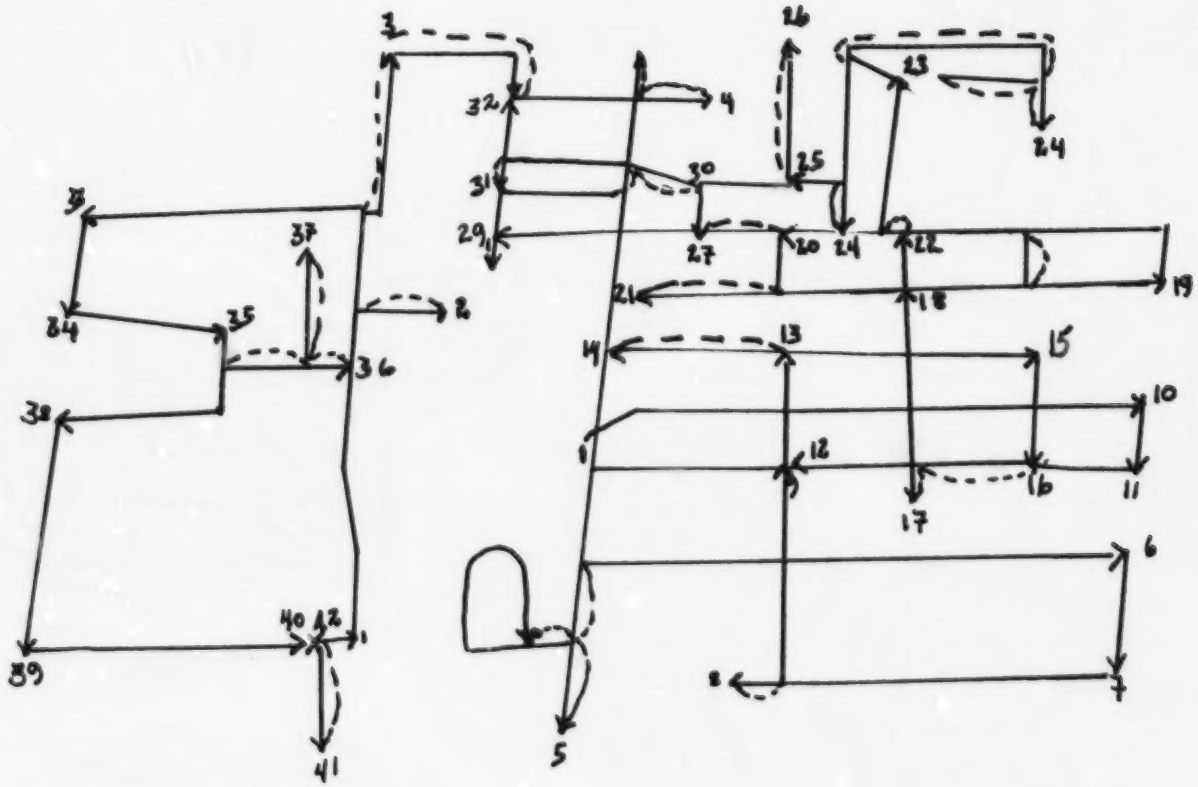
—— North York Roads

o Odd Nodes

═══ Metro roads partially included on the basis of minimum distance to be travelled and minimum number of odd nodes created.

Figure 12

EULER PATH FOR INROAD 4



--- Duplicated Links

Total Distance 14.2 miles

Table 1

SOLUTIONS FOR THE MINIMUM-DISTANCE PATH USING INTUITION FOR INROAD 4

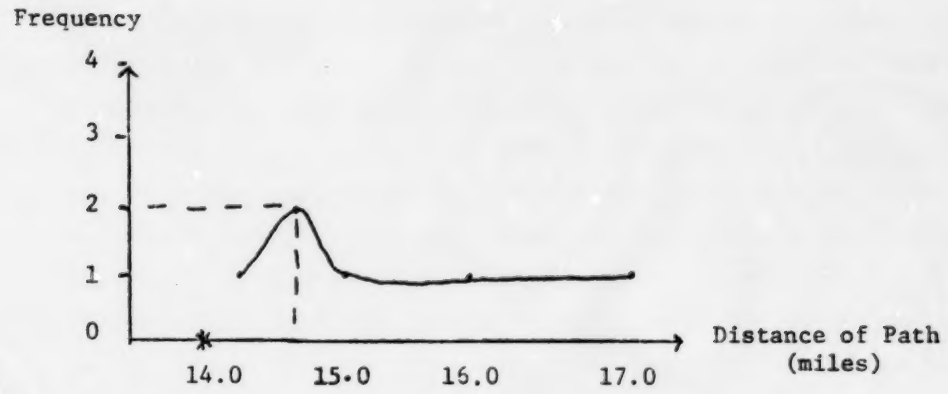
Subjects	North York Roads (constant) max. = 10.3 miles	Metro Roads (optional) max. = 2.6 miles	Duplicated Metro & North York Roads max. = 12.9 miles	Total Distance	Percentage Difference From Optimal
1	10.3 miles	1.1 miles	2.9 miles	14.3 miles	0.7%
2	"	1.2 "	3.2 "	14.7 "	3.5%
3	"	1.7 "	3.1 "	14.7 "	3.5%
4	"	2.0 "	2.7 "	15.0 "	5.6%
5	"	2.0 "	3.6 "	15.9 "	12.0%
6	"	2.6 "	4.2 "	17.1 "	20.4%
				AVG.	7.6%
OPTIMAL (Euler procedure and intuition)	10.3 miles	1.0 miles	2.9 miles	14.2 miles	

optimal path is 14.2 miles. The percentage increase in the value of the total distance derived from using intuition when compared to the optimal value ranges from 0.7% to 20.4%.

With the values for the minimum-distance path generated by the six subjects, we can draw a frequency curve which has the form shown in Figure 13. For the sample size used in this experiment (where n = 6) the form of the frequency curve is a positively skewed distribution. If the sample size n is larger, a positively skewed form of distribution would probably still be obtained. The inference of the positively skewed distribution is that the majority of the solutions achieve near optimum values. The hypothesis that is being proposed is that the positively skewed form of the distribution curve does not change as n increases. This hypothesis can be argued to be true because the solutions that are being made are not random solutions but solutions with a specific objective, ie., minimizing the distance of path. If random paths were being selected, then a more normal distribution would result, though the form of the distribution is related to the geometry of the network. A similarly skewed distribution results from the 20 solutions for Monroe's location problem (see Massam 1975, pp. 92-95). The responses indicate that 50 per cent of the intuitive solutions selected the optimum pair of locations. One of the conclusions of an experiment carried out by J.B. Schneider was "that a relatively inexperienced, but intelligent, person can rather quickly solve a fairly simple location problem nearly as well as one of the better computer techniques now available" (Schneider 1971, p. 99). In his report, Schneider presents an experiment designed to compare the ability of human intuition with that of a computer in the selection of a near-optimal location pattern for ambulance dispatch centers in a hypothetical urban area. As such, this report demonstrates the effectiveness and flexibility of human approaches, and indicates a possible strategy for future planner-computer collaboration (Schneider 1971, p. 95). Therefore, we could argue that intuition can serve as a useful method for solving location and/or routing type problems.

Figure 13

FREQUENCY CURVE OF MINIMUM-DISTANCE PATH SOLUTION



* Optimal Path
n = 6 subjects

3.0 CONCLUSIONS

Several conclusions can be drawn with respect to the two methods for identifying a minimum-distance path.

One conclusion is that the Euler path procedure can be significantly better than intuition. The average total distance of the minimum-distance path using intuition is 14.5 miles. Compared to the Euler path procedure, intuition produces, on the average, 2.1% more for the minimum path to be traversed through Inroad 4 street network. This difference may imply that intuition, on the average, does not produce significantly larger values for the minimum-distance path compared to the Euler method. Although the sample size of solutions using intuition is small, it clearly indicates that intuition can be used to identify paths that are close to the optimal--if not the optimal--solution.

A second conclusion concerns the analysts who identified the paths by intuition. The lowest values for the minimum-distance path are derived by those three analysts who have had previous exposure to the nature of street networks (Subjects 1, 2 and 3). Thus, the abilities of the analyst to recognize certain characteristics of a street network influence the values of the total distance of the minimum-distance path.

A third conclusion is that the Euler method allows for early identification of the odd nodes, whereas intuition requires a number of attempts (depending on the analyst) before those links that have to be traversed more than once are identified. Although the Euler method requires less time for identifying links to be duplicated, this characteristic is impeded when the analyst performs the task of identifying a Euler path through the street network. The identification of a Euler path, like intuition, is based on trial and error.

A fourth conclusion is that the Euler method is only applicable for street networks that are undirected and require a single pass. For the

snowplowing operation, a real street network is directed and requires two passes in opposite directions for each street. Hence, the Euler method requires modification in order to apply it to a real street network.

A fifth and final conclusion is that the identification of a Euler path depends on the complexity of the street network to which it is applied. The connectivity of the links, the complexity of the pattern of the streets, whether there are several Euler paths or only one, the location of odd nodes, etc. all influence the preference of the Euler method over intuition for identifying the minimum-distance path. Schneider argues for a combination of an algorithm and intuition as an efficient strategy for tackling more complex problems typical of large metropolitan areas (Schneider 1971, p. 99).

PART TWO:
EULER'S PROCEDURE AND INTUITION AS A COMBINED FORMAL METHOD FOR SOLVING MINIMUM-DISTANCE PATH PROBLEMS (ONLIP)

1.0 GENERAL INTRODUCTION

Vehicle routing problems have a recurrent theme: to minimize the total distance travelled in traversing a path through a network, ie., in traversing every edge or link in the network at least once. Stricker's (1970) survey of existing solution techniques for the problem provides an excellent summary of various ways to tackle edge-covering problems. The various research on vehicle routing problems indicates a consensus among researchers--whether they are in the field of computer science, geography, transportation, engineering or operation research--that there is not, at the present time, an acceptable algorithm to solve a practical-sized problem, or a problem with additional constraints such as street priorities. In fact, most real world edge-covering problems are solved by heuristic techniques or simply by modifying a path that has been covered in previous years. The evaluation of all feasible alternate paths is prohibitive. Because of the high costs incurred for public services, there is more demand for research into more efficient algorithms or techniques to solve edge-covering problems. This type of research will enable management to make decisions related to the routing pattern to reduce the costs of servicing a network and at the same time increase efficiency.

This section focuses on this branch of minimum-distance path studies where a comparison of intuitive versus a systematic procedure is undertaken. It would seem that an interface system is most appropriate, whereby the Euler method is used to define and link odd nodes, and the analyst searches for the minimum-distance path using his/her intuition.

The first half of this section introduces the Euler and the intuitive procedures as two separate solution methods for solving the vehicle routing

problem. The second half outlines, in detail, three experiments (prepared by the author) to test the new procedure. The objective, the method, and the details of the subjects for the experiments will be discussed. The presentation and examination of the results of the experiments follow this discussion. Finally, concluding remarks on the Euler method and intuition as a separate and as a combined formal procedure for solving vehicle routing problems will be made. Hence, the aim is to provide some insights into the applicability and limitations in applying the ONLIP (Odd Node Linking Intuitive Procedure) in identifying the minimum-distance path for vehicle routing problems.

2.0 EULER PROCEDURE

A Euler tour is a continuous path through a street network such that each edge is covered exactly once. The governing assumption of the Euler method is the existence of 0 or 2 odd nodes. If this condition is not present in a network then odd nodes are paired in order to identify those links that are to be traversed more than once. Whenever routing problems for urban service systems are posed in a geographical context, then the manual approach can provide solutions for matching odd degree nodes by inspection with the aid of a good map. In the manual search for a minimum-length matching of odd-degree nodes, no two shortest paths in the matching can have any edges in common. Once the network has been expanded to a Euler network, the analyst proceeds to derive a Euler path by starting at an odd node (or any node, if there are no odd nodes) and erasing each street as it is covered but not erasing a street which would divide the remaining network into two unconnected parts.

In a real street network, the number of ways to pair odd nodes can become difficult to calculate; 50 odd nodes calls for 1225 separate calculations.

Many researchers who have applied themselves to this problem have devised ways of choosing the odd nodes to join together which cut down the

theoretically necessary number of calculations. But even these methods are not very efficient for large problems. In fact, the quickest and most satisfactory method (unless the procedure has to be repeated frequently) is to do it by eye: to mark each odd node in the street network and link them by hand (Transportation Development Agency 1975).

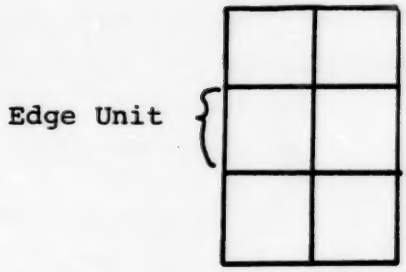
The first part of the Euler method consists of a manual but systematic procedure that is the identification and linking of odd nodes, whereas the second part, finding an Euler path, depends on the pattern recognition abilities of the analyst.

The solution method for dealing with any street network of real size depends on the pattern recognition abilities of the analyst. Experience has shown that the necessary skills are quickly acquired with little practice (Transportation Development Agency 1975).

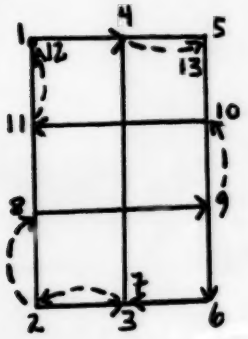
Hence, executing the latter part is a trial and error process which usually reflects the degree of skill or experience with networks. In solving for the minimum-distance path, analysts who repeat the exercise using the same network sometimes produce different types of paths which may vary in both the length of the duplicated edges as well as in the pattern of the route. Figure 14 presents an example of two different patterns resulting in 5 units and 3 units of duplicated edges in the 'U-Shaped' and the 'Box-Shaped' patterns respectively. An analyst who can identify a minimum-distance path that is close to the optimal path would be one who has insight or a 'talent' for recognizing a pattern in a street network. This type of ability will be referred to as intuition. One who has an intuitive talent has the ability to recognize and employ various routing or location choices in order to reach the objective; the objective may be to minimize total path distance, minimize total cost, minimize travel time to destination point, and the like. This talent can be developed by repeated exercises of various location or routing-type problems so that the eye can be trained to recognize the variety of possibilities for solving such problems.

Figure 14

DIFFERENT ROUTING PATTERNS

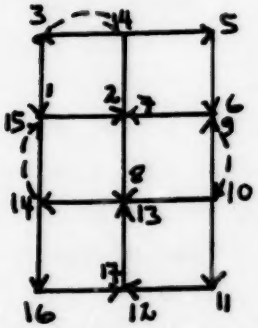


Network
To Be Traversed



'U-Shaped'
Pattern

(5 units duplicated)



'Box-Shaped'
Pattern

(3 units duplicated)

3.0 INTUITIVE PROCEDURE

Perhaps most important, the graph-theoretic approach, based as it is on the pictorial representation of problems in a network context, lends itself in an excellent way to an intuition-based presentation of algorithms and to the discovery of simple, ad hoc procedures for obtaining good approximate solutions to difficult and mathematically complex problems (Larson and Odoni 1981, p. 393).

For application in urban service systems, the graph-theoretic approach is a most natural one since an urban transportation grid can readily be translated into a network model.

> ...algorithms are also described as exact or heuristic. The former are guaranteed to terminate, given sufficient (computer) time, with an optimal solution to the problem which they are designed to solve. Heuristic algorithms do not offer such a guarantee; they usually trade off "guaranteed optimality" for speed in providing an answer (Larson and Odoni 1981, p. 379).

The intuitive approach does not depend directly on computing the cost of alternative paths through a network in a rigorous fashion but rather depends upon an individual's ability to treat the problem visually. The intuitive answers can be compared to those generated by a systematic procedure, and thus an evaluation of the utility of either the intuitive approach alone, or in conjunction with the Euler procedure can be made. Moreover, an evaluation of the attributes of different groups of individuals in relation to their intuitive skills in solving least-distance path problems can be carried out. This will be the subject of discussion in the following sections. This is an important consideration if public sector routing problems are to be solved by people trained to make better intuitive decisions.

The study of intuition as a method for problem solving is well documented in the field of social psychology (Kaplan and Schwartz 1975). On the one hand, intuition and professional judgement derived from past experience are of demonstrative utility in solving minimum-distance paths for complex road networks and are frequently used both to choose the path to be traversed by a vehicle and to defend the choice. On the other hand, a stronger case can be made if evidence showing the relative merits and drawbacks of alternative paths is available, and the solution is more easily scrutinized by the public, policy makers, professionals, and operators of vehicles who traverse the paths.

> The reliance on heuristics and the presence of common biases are general characteristics of intuitive judgement under uncertainty. They apply not only to laymen untutored in laws of probability, but also to experts--when they think intuitively. (Quote by Tversky and Kahneman 1973 in Massam 1980, p. 78).

Although the above quote refers to individuals choosing a preferred alternative site for a facility, its application to choosing alternative paths in a network can easily be seen.

4.0 MINIMUM-DISTANCE PATH SOLUTIONS

Before proceeding with the examination of the results from the three experiments, the objectives, methods and subjects will be discussed.

4.1 Objectives

The three experiments are designed to meet the following three objectives:

1. To test if the intuitive method produces lower minimum-distance values with the same subjects over time
2. To test if various types of subjects produce significantly different values for the minimum-distance path

3. To test if intuition, when aided by the Euler theorem, produces significantly lower values for the minimum-distance path.

4.2 Method

The method of the experiments entails the following:

1. The selection of subjects who are willing to take time working with a network

2. The selection of a street network that is challenging but not overwhelmingly complex, that is undirected, requires single passes, and has no constraints (i.e., traffic direction, multiple passes and street priority constraints)

3. The preparation of instructions which are included in an experiment package, containing a copy of the street network to be solved, an example of a solved street network, and several sheets of onionskin paper for tracing the solution path (refer to appendix)

4. The distribution of the experiment package, which often requires a follow-up for those subjects who may require assistance in completing the assigned task

5. The calculations of the duplicated edges only, because all links in the street network had to be covered, therefore they are considered to be a constant in the calculations. By using a network with all the links pre-measured (in units), the distance for duplicated edges is easily calculated with minimal error possibility (refer to Table 2)

6. The analysis of the results.

4.3 Subjects

The three experiments were given to three groups of subjects with different occupational backgrounds. The first group of subjects have a

background in the field of geography (ie., geography graduate students, or one in a geography-related profession) or have had previous exposure in the field of graph theory (Group A). The second group is made up of civic employees specifically, the superintendents and plow operators for the snow-plowing routes in the City of North York (Group B). The third group of subjects are labelled 'laymen' (Group C) and their occupation backgrounds vary (ie., secretary, clerks, teacher, salesman, barber, etc.). The subjects were selected and placed in a group using the author's discretion.

The author realizes that the sample size is small--seven subjects in each group with a total of twenty-one subjects--and that only one type of network is used bearing no constraints of the type present in real world vehicle routing problems. Due to the time constraint, subject availability for network-type experiments and costs, it was not possible to undertake a large-scale experiment. Moreover, the nature of the experiment requires constant contact with some of the subjects, who tend to run into difficulties. Hence, it was only possible to be available for a small sample size. The street network that the subjects worked with is taken directly from one of the two sample problems presented in the Transportation Development Agency report (1975) and was given to subjects in Group C. The subjects in groups A and B were given the same street network but one lacked the identification and pairing of odd nodes. Both types of networks did not consider street priority, traffic direction and multiple passes constraints. Rather, the focus was purely on the vehicle routing pattern where the objective was to cover the street network while minimizing multiple passes. Hence, it is the author's opinion that the design of the three experiments was appropriate for achieving the three objectives: provide a good indication of how different subjects solve this type of street network with increased number of trials, with different degrees of experience in the field of graph theory and with Euler's theorem accompanying intuition.

4.4 Results of Experiments

Tables 2 and 3 present the calculated values of street duplication for the street networks solved by each subject that took part in the three

Table 2

DUPLICATION OF MINIMUM-DISTANCE PATH (IN UNITS) BY SAME SUBJECTS WITH SUCCESSIVE TRIALS USING INTUITION AND WITH EULER'S THEOREM

Subjects (Group A)	Trial #1	Trial #2	Trial #3	Euler's Theorem and Intuition
1	56.9 (units)	45.4 (units)	43.8 (units)	45.2 (units)
2	92.4 "	95.2 "	76.1 "	53.7 "
3	44.0 "	44.4 "	44.4 "	44.0 "
4	60.6 "	52.6 "	52.4 "	58.3 "
5	83.4 "	77.7 "	67.6 "	52.1 "
6	58.6 "	74.1 "	63.0 "	53.7 "
7	53.8 "	67.9 "	73.7 "	52.7 "
Average	64.2 (units)	65.3 (units)	60.1 (units)	51.4 (units)
Range	48.4	50.8	32.3	14.3

Table 3

DUPLICATION OF MINIMUM-DISTANCE PATH OF DIFFERENT SUBJECT GROUPS USING INTUITION (In Units)

Subject Number	Group A (Geography Related)	Group B (District Superintendents)	Group C (Laymen)
1	43.8 units	52.5 units	60.7 units
2	76.1 "	50.4 "	60.8 "
3	44.0 "	82.3 "	66.3 "
4	52.4 "	81.1 "	59.8 "
5	67.6 "	68.4 "	65.9 "
6	58.6 "	55.6 "	61.3 "
7	53.8 "	60.6 "	64.8 "
Average	56.6 units	64.4 units	62.8 units
Range	32.3	31.9	6.5

experiments. Since the sample network is not a real one without a scale, distances for the duplicated edges are in units.

When the Euler path is solved for the sample network the duplication is 51.8 units. The intuitive method by the same subjects who attempted to solve the minimum-distance path three times had mixed patterns of improvement. The units of duplication of the street network by the seven subjects indicate that three sets of solutions had decreasing values for each successive trial (Subjects 1, 4, and 5) and the other four subjects had fluctuating values for duplication of streets with each additional attempt at obtaining a lower value for the total distance of the path.

The reason for the difference in the improvement pattern is related to the subject's pattern recognition ability. The question of how the experiment was tackled by each of the subjects influences the improvement pattern for the three trials. Those subjects (Numbers 1, 4 and 5) who obtained lower values for the minimum-distance path in the consecutive trials applied the same specific pattern, ie., diagonal wedges, loops covering the entire NS and EW edges, zig-zagging across, etc. Subject Number 3 also used this method but did not significantly reduce the minimum-distance value. Those subjects (Numbers 2, 6 and 7) who had fluctuating improvement patterns for the minimum-distance path for the successive trials tackled the street network by applying different strategic patterns, or no pattern at all for covering the street network.

For the same street network three groups of subjects produced different values for the minimum-distance path: the average length of the duplications of the edges for Group A (geography-related subjects), for Group B (District Superintendents) and for Group C (laymen) are 56.6 units, 64.4 units and 62.8 units, respectively. The range for the minimum-distance values in each group is lowest for the laymen with 6.5 units and for the other two groups the range is approximately 32 units. The lowest values for the minimum-distance path are obtained by subjects Numbers 1 and 3 in Group A with values of 43.8 units and 44.0 units, respectively.

When intuition is aided by the Euler theorem, four out of seven subjects produced significantly lower values for the minimum-distance path, two subjects generated larger values (Numbers 1 and 4), while one subject (Number 3) reproduced the same lowest value as in the first trial. It would seem that the majority of the subjects are aided by the Euler theorem in reducing the duplication values for the minimum-distance path.

4.5 Costs

If this field of research is to assist the provision of public services, then it has to be linked to cost. The quantity and quality of a public service is directly affected by the alloted availability of revenue, eg., the budgets for police and fire protection, health and welfare, education, refuse collection, and snow removal.

Ideally the cost of snow removal would be measured in dollars per mile of road cleared. If it is possible to reduce the distance of the route without reducing the quality of the delivery of a service, then financial savings can be identified. In order to relate distance to cost, the cost figures for the snow removal operation for the City of North York are considered. Unfortunately, an analysis of costs and distance for the snow removal operation for this jurisdiction are not available due to the lack of disaggregated data for this operation.

An attempt to analyze the relationship between cost and distance is made using only wage rates of snowplow operators. When one considers the cost of snowplowing in relation to path distances, a gross simplification of the calculations is required where all other important parts of the snow removal operations are excluded (eg., sanding, salting, management, patrolling, equipment). One factor that varies with the distance of path is the time taken to cover the path. The rationale here is that the longer the snow removal path the longer the vehicle has to be operated. Since the operators of roadplows in the City of North York are paid for the time spent on plowing, the costs of snow removal are reduced when the distance (in miles)

of the snow removal route is reduced. The proposed hourly winter labour rate for the City of North York for the 1981/82 season is $9.95 per hour. An average of 10 hours is required to plow one route.

For the real street network in Inroad 5, for example, the average route mileage is approximately 11.0 miles. To clear the street network in Inroad 5 a total labour cost of approximately $99.50 would be incurred and it would cost approximately $9.04 per mile of snow cleared.

Suppose the path of the snowplowing route in Inroad 5 is shortened by 15.3%, then the new total mileage of the path would be 9.8 miles. Assuming that the cost per mile remains the same, then the total cost for plowing the street network would be $88.59; a cost saving of approximately 11% (or $10.91). Based on the possible reduction in path mileage from the previous experiment, a reduction in snow removal costs can be significant.

5.0 CONCLUSIONS

Both the intuitive and Euler procedures contribute in a systematic way to determining the minimum-distance path in a network. The Euler procedure identifies the odd nodes and the links to be covered more than once while the intuitive approach identifies the direction of the path to be traversed through a network.

As a combined formal procedure, the Euler method and intuition (ONLIP) is a dynamic procedure that allows the minimum-distance path to be adjusted when changes are made in a street network or when modifications in the operation of a public service are made, eg., budget, policy, number of equipment, labour, routing pattern, and size of snow plowing district.

Although the experiments included a small sample size (21 solutions) the results for the duplication values indicate a representative sample.

Those subjects that have been previously exposed to solving network-type problems are able to apply various strategic routing patterns for identifying the minimum-distance path. In fact, Subjects 1 and 3 in Group A and Subject 2 in Group B each solved a path through the street network with unit values for duplication of 43.8, 44.0 and 50.4, respectively. These three path solutions have lower values for units duplicated than the 51.8 units of duplication required in solving for the Euler path in the same street network compared to 43.8 units of duplication as indicated by the solution produced by subject no. 1 in Group A. The distance of the path can be reduced by a maximum of 8.0 units, reducing the distance of clearing the roads in the sample street network by 15.3%. The possibility of implementing the routing pattern of these paths for a real street network has not been considered and is subject to much more work.

In sum, the intuitive approach is likely to yield a path similar to the optimal path provided by the Euler procedure, and a combination of both approaches insures the optimal path while less computing costs are incurred. Finally, this particular field of study--intuitive approach and Euler procedure (ONLIP)--demands further work. More experiments of the type carried out in this paper could be done for snow removal routes in different locations and with a larger sample size for the subjects.

PART THREE:
APPLICATION OF ONLIP TO A REAL STREET NETWORK

1.0 GENERAL INTRODUCTION

Up to this point the research has focused on tackling vehicle routing problems on street networks that have been simplified for the purpose of testing and analyzing theoretical (Euler's theorem) and manual procedures. The application of the ONLIP is satisfying on an intellectual, basis but its strengths and weaknesses, on a practical basis, remain to be discussed. This section takes the ideas that have already been developed and applies them to improving the snowplowing routes in a real street network.

2.0 ACTUAL PATH

The path followed by snowplows in the City of North York should conform to two constraints, namely, street priorities and directed double passes. The distance of the path presently traversed by snowplow operators for Inroads 4 and 5 cannot be compared to the distance value of the path produced by applying Euler's procedure or intuition or the combined method of the latter two, because they do not consider these constraints, there is no street priority and no directed multiple passes. The snow removal operation in the City of North York, as in many other jurisdictions, cannot be described as a systematic, static or clear-cut operation simply because the nature of winter maintenance places city management officials in the hands of climatic whims. Moreover, parked cars, traffic, salt and sanding operations during icing conditions and other obstructions on streets, when present, require management officials to make on-the-spot decisions with respect to altering the path and/or adjusting the snow removal operation so that quality snow removal service is delivered. During storms or unusually harsh weather conditions a snowplow operator may complete the 'opening-up' (initial pass) of the red routes only; in milder snow falls, it may require the same time to clear all the streets in his designated route. Hence, it is fine

to say how a route should be traversed in theory, but in reality various factors alter the smoothness of the operation.

There is yet another complicating factor in comparing the two types of distance values, and that is the incorporation of the travelled distances as opposed to the plowed distances. The snowplow travels (not plows) on those North York roads that have been cleared of snow and on Metro roads for which the said jurisdiction is not responsible for clearing. The distance of an edge whether it is travelled or plowed remains the same, but the time and costs incurred by travelling the same edge is less than that incurred by plowing. Thus, travelling, as opposed to plowing, is another factor that must be taken into account when comparing minimum-distance values for a sample network with a real network.

Due to these major differences--street priority, directed double passes and travelling--between sample networks and real networks, the minimum-distance values for Inroad 4 using the combined Euler and intuition procedure (ONLIP) cannot be directly compared to the distance values for the paths traversd in reality. A close inspection of how the contractors for the City of North York presently route the snowplow through the real street network of Inroad 4 and 5 results in the detection of major differences compared to the routing pattern derived using ONLIP with the simplified network for the real street networks (refer to Figs. 15 and 16).

For the single pass hypothetical network there were a number of odd nodes produced, but for the real network where double passes are required, the ends of edges are joined to either the North York roads or Metro roads thus allowing for no odd nodes to be produced. In the City of North York one-way streets are not found as they are in older areas like the City of Toronto. A network that has one-way streets would allow for the identification of odd nodes and by inference streets that must be traversed twice. Since Inroads 4 and 5 do not contain any odd degree nodes, then the street networks each possess an Euler path. The only indication where the

Figure 15

INROAD 4

Direction of Real Path

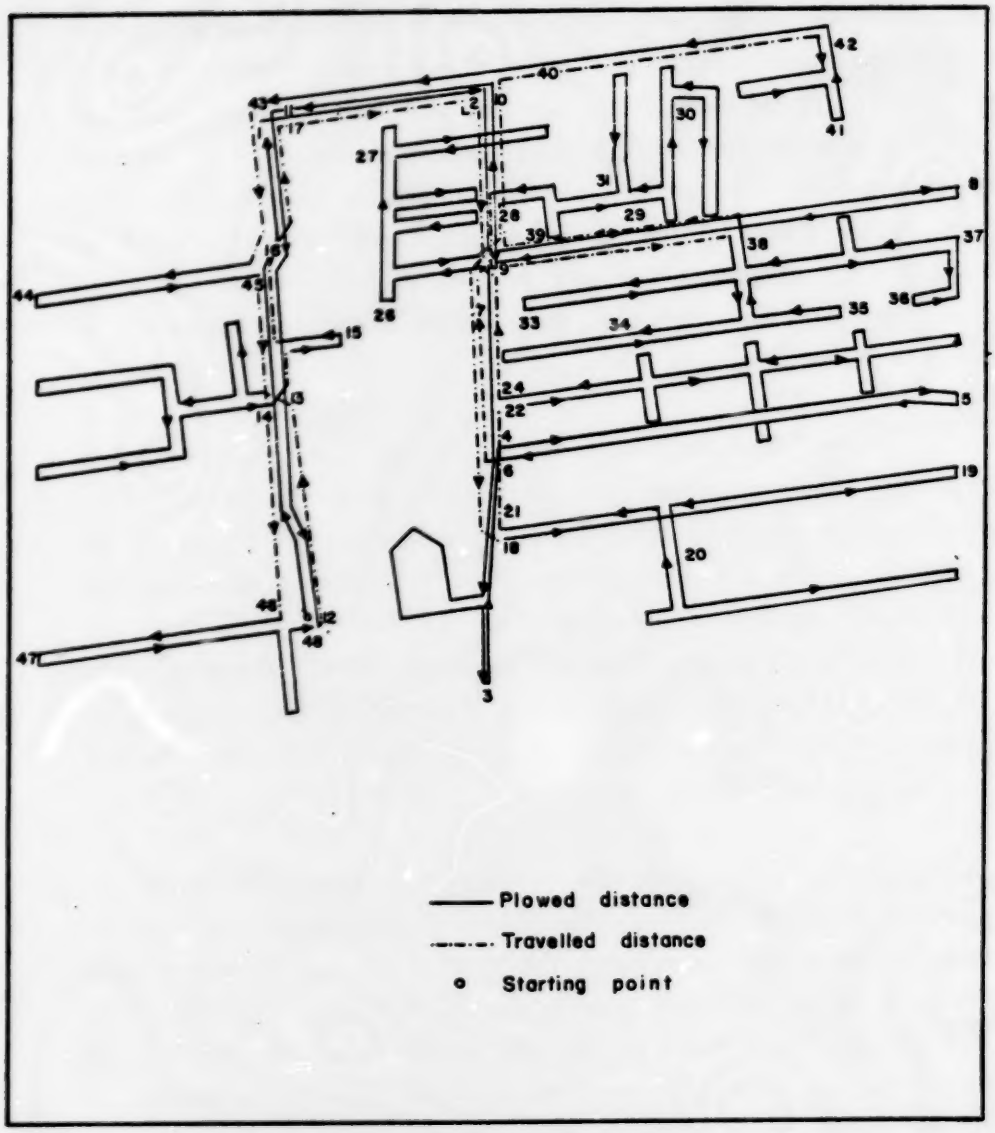
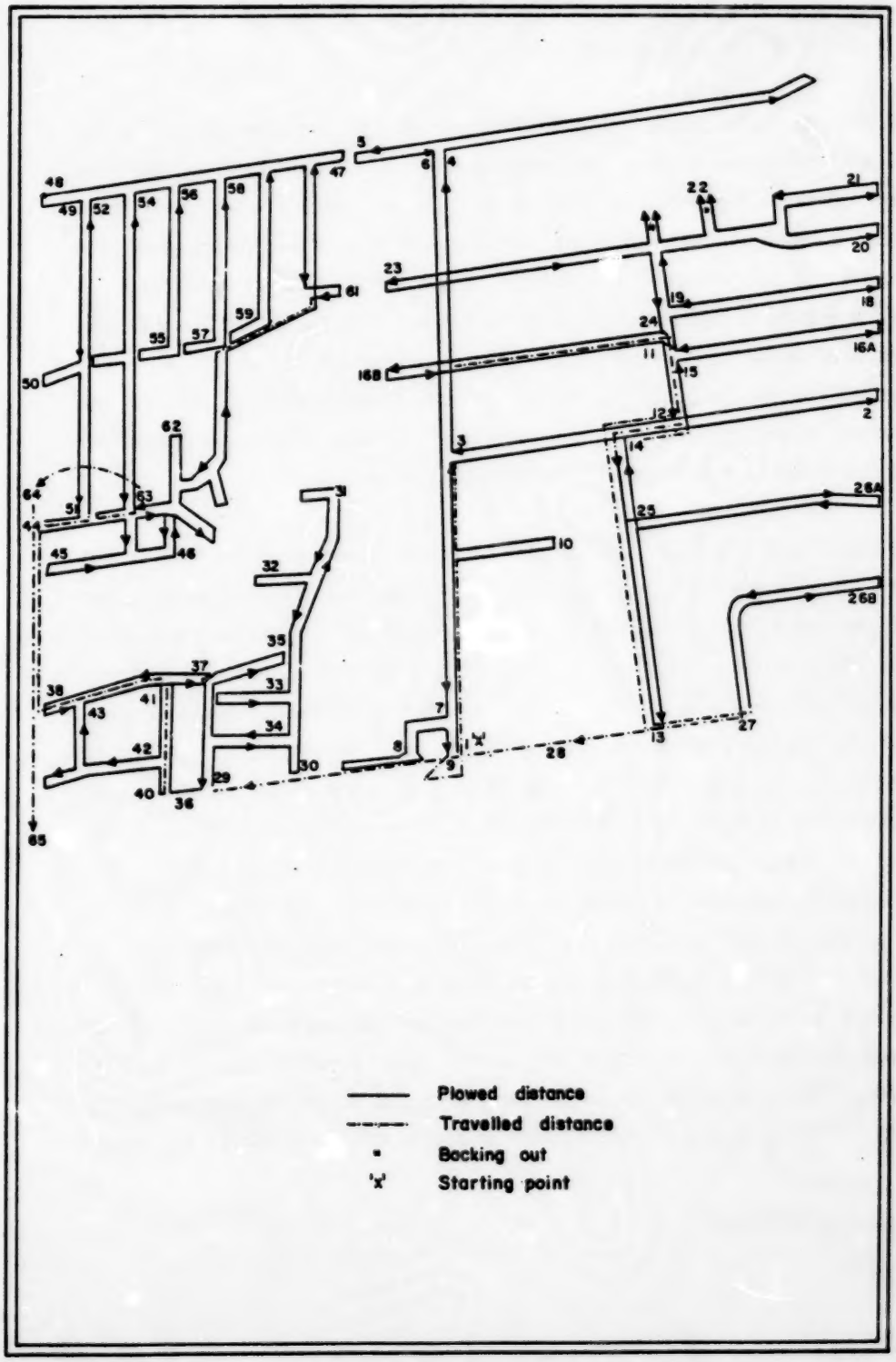


Figure 16

INROAD 5

Direction of Real Path



direction of an Euler path would not be followed is in those narrow dead-end streets that do not have a cul-de-sac with a large radius which would enable the snowplow to turn around. Two such streets where the snowplow cannot turn but backs in and pulls snow out are in Inroad 5 (refer to (") in Fig. 16) so that each street requires two passes.

With the odd nodes being excluded from the snowplowing paths for Inroads 4 and 5, the problem of reducing edges that must be duplicated no longer exists. The problem that remains to be considered is the street priority constraint. The present direction of the snowplowing path for Inroads 4 and 5 is designed to conform to the street priority constraint. A close inspection of the actual path reveals that the majority of the time red routes are plowed first and yellow routes are usually combined with the unmarked routes. In other words, for Inroads 4 and 5 the red, yellow and unmarked routes are not plowed separately in their respective order, but there is a definite indication of an attempt to do so.

There are a number of reasons why the street priority rule is not obeyed all of the time. One reason is to increase efficiency; covering one unmarked short street located in an area of yellow routes or red routes means more time, distance and costs incurred when done separately compared to amalgamating the unmarked street into the path that covers one of the marked routes. A second reason may be that, although a map indicates that a street has been placed in a priority category, the street situation, such as employees of an industrial area parking on one side of the street, may force the street to take lower precedence until such time as the vehicles are removed so that the street is plowed with the unmarked routes or at a later time. During heavy snow falls, when plowing speed is at its minimum, unmarked routes may not be cleared because the climatic conditions keep snowplow operators busy just keeping the red routes 'opened' for vehicular travel. In such cases the yellow and unmarked routes can be considered to have no priority. A fourth reason for combining different priority streets in a single priority path is due to the direction of the snow-

plow. Snowplows do not have the capacity to contain snow as it is being cleared, and oftentimes citizens complain about the dumping of snow on their driveways or boulevards. City of North York officials, in an attempt to respond to this complaint, aim to maximize right-hand turns and minimize travelling in a straight line for long distances in order to distribute access snow that the blade on the snowplow cannot contain in more than one place.

3.0 APPLICATION OF ONLIP

The questions that arise at this point are related to the application of ONLIP. So far, this paper has described various problems in applying the results of the simplified model to a real life situation. It would be interesting to go one step further by outlining how one would apply the results of this study. It was not possible to carry out an experimental design that would either confirm or invalidate the tentative research results on the basis of an application of ONLIP to a real street network with all its real-life complexity. However, such an experimental design will be described.

The steps involved in the application of ONLIP to a real street network are:

- begin with identifying odd nodes for a partitioned network
- link pairs of odd nodes, thus identifying streets which must be traversed twice
- trace the minimum-distance path by using Euler's theorem
- identify obstacles (parked cars, traffic regulations, different priority streets to be integrated, etc.) on real street network
- retrace minimum-distance path and the ONLIP-generated path so that decisions regarding revisions to the existing routing pattern can be considered.

For the City of North York, for example, the steps for the application of ONLIP require the cooperation of each of the six District Superintendents and the operator(s) who are familiar with the designated areas.

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Appendix
ORGANIZATION OF EXPERIMENTS 1, 2 AND 3

For experiments 1 and 2 the subjects were instructed as follows:

EXPERIMENT NO. 1: INTUITION (Same Subjects Over Time)

Objectives To traverse every link in the street network once and only once, so as to minimize total distance of the path to be traversed by one vehicle through the street network.

Procedure
1. Begin and end at the same location on the street network to be identified by "X".
2. Where a link is traversed more than once, indicate by a dashed line.
3. You are to solve a path (or a tour) through the street network three times in order to see if there is an improvement in making more than one attempt at solving a street network (refer to Sample Street Network A).
4. Number your solutions in the order of completion.

EXPERIMENT NO. 2: INTUITION (Different Subjects)

Objective Same as Experiment one.

Procedure 1 and 2 remain the same as in Experiment one.
3. Your attempt at solving the street network is to take no more than one hour. Please submit the solution that you believe is the least-distance path (refer to Sample Street Network B).

EXPERIMENT NO. 3: EULER'S PATH AND INTUITION (Subjects Related to Geography)

Objective Same as in Experiment One.

Procedure 1. Same as in Experiment One

2. Where a link is to be traversed more than once it already been identified by the dashed lines (refer to Sample Street Network No. 2). Should there be any other links to be traversed more than once, please indicate this by tracing the link in red.

In addition, the subjects were all given an example of a street network along with its solution and several sheets of onionskin paper for tracing their solution.

POOR

QUALITY

ORIGINAL